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STEAME ACADEMY TEACHING FACILITATION LEARNING & CREATIVITY PLAN (L&C PLAN) - LEVEL 2 STUDENT TEACHERS: Harmony – the Beauty of Mathematics

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1. Overview			
Title Driving Question or Topic	Harmony – the Beauty of Mathematics. Mathematics in Arts and Nature. How is harmony defined in Mathematics and in the Arts (visual arts and music)? How mathematical concepts work at the heart of what we find pleasing to the eye and the ear? What is the golden ratio ("the divine proportion", "proportion of beauty") and where it can be found in the shapes of living organisms, paintings, sculptures and buildings? Is Mathematics the language of nature?		
Ages, Grades,	14-15 years old	8-9 grades	
Duration, Timeline, Activities	18 lessons	18 classes each with duration of 40 min.	1 or 2 classes per week for 9-18 consecutive weeks
Curriculum Alignment	Mathematics, Art (visual art, music), Biology, IT (Graphic Design) Learning the mathematical basis of the theory of harmony – from the school of Pythagoras, Platonic solids and the geometrical definition of the golden section by Euclid, through the contributions of famous European mathematicians like Leonardo Fibonacci, François Lucas, Luca Pacioli, Jacques Binet, Jacob Bernoulli, Johannes Kepler and others to the more recent generalizations of the golden ratio called metallic means by Vera W. de Spinadel. Learning how these fundamental mathematical concepts are found in artworks, design and architecture, the cosmos and the living nature. The importance of Mathematics as the ultimate mean to understand and shape the world around us.		
Contributors, Partners	Art galleries, History museums, Graphic design studio		
Abstract - Synopsis	Initially, the students are taught by a math teacher who introduces them to the mathematical definition of harmony and its geometrical aspects. The math teacher follows the historical development of the concept of harmony explaining and underlining its close ties to the arts and study of the universe and the living nature. First, the students learn about the golden section, its geometrical definition and that it is the positive root of a certain square equation. Then, the students learn about some generalizations of the golden section – the silver section and other metallic means, how they can also be defined as roots of square equations of these irrational numbers to famous		

	number series, like the Fibonacci, Lucas and Pell numbers. The students solve geometrical and algebraic problems involving the golden and other sections. A biology teacher explains to the students where the golden ratio as the Fibonacci spiral (logarithmic spiral with the golden ratio as the growth factor, i.e. the spiral of growth) are found in various shapes of living organisms. The art teacher (visual arts, preferably with experience in graphic or logo design) introduces the students to the concept of harmony in the arts, to famous works of painters (for example Leonardo da Vinci, Salvador Dali) and sculptors (for example Phidias), artifacts and buildings from ancient times to modern days which are based on the golden section or silver section. An IT teacher or the art teacher introduces the students to some basic functions of computer software which can be used for graphics and logo design, like Adobe Photoshop, Illustrator, Corel Draw, Wix Logo Maker, Canva, Adobe Express, Ucraft, GIMP or others. (Optional) A music teacher explains to the students how the concept of harmony in music is based on the same principle as in mathematics – ideas going back to the school of Pythagoras about the harmonic series, musical intervals and musical distances which sound pleasing to the human ear. The students can browse the Internet and/or visit art galleries and museums together with the math and the art teacher to look at various works of art and discuss the golden ratio and other harmony relations presented in them. Next, with the assistance of the school management, a meeting with a professional graphic designer is organized. The graphic designer also explains to the students how to use software to design a simple logo or a simple digital artwork and how to incorporate the concepts of harmony to make it more aesthetic. The designer presents to the students some of their artworks or works by other designer presents to the students some of their artworks or works by other designer presents to the student some of using i
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2. STEAME ACADEMY Framework^{*}

Teachers' Cooperation	 Teacher 1: Mathematics Teacher – explains to the students the basic mathematical theory of harmony in geometry and algebra. Teacher 2: Art Teacher (visual art teacher, preferably with some experience in graphic or logo design) – explains to the students how the theory of harmony is applied in the visual arts by introducing them to artworks of painters, sculptors, architects and historical artifacts in which the golden section or other metallic means can be found. Teacher 3: IT Teacher – an IT teacher (or the art teacher) explains to the students how a basic graphic design software can be used for simple logo design. Teacher 4: Biology Teacher – explains to the students how the golden ratio holds a special significance in nature because it is found in various living forms, including the human body. (Optional) Teacher 5: Music Teacher – explains to the students the basic principles of Western musical harmony – intervals, harmonics (overtones), triads, fifths, chords (triads – major and minor triads, tonic, dominant and subdominant chords), chord progressions and how to harmonize a simple melody. Also explains how the golden ratio has been used by composers to evoke a sense of harmony and satisfaction to the listeners.
STEAME in Life (SiL) Organization	Meeting with professional graphic/logo designers. Visit to a graphic design studio.
Action Plan Formulation	 Step 1. Acquisition of theoretical knowledge. The students learn about the theoretical principles of harmony in mathematics and how they are applied in biology and the arts – visual arts and music. Step 2. Acquisition of computer skills. The students learn about the basic functions of graphic design software and how to create a simple logo or digital artwork by its help. Step 3. Discussions and inspiration. The students discuss among themselves and with the art and math teachers about the presence of harmony, mainly the golden and silver ratios, in various shapes created by man or nature, visit art galleries and museums, and analyze digital artwork found on the Internet. A professional graphic designer introduces the students to digital works of art, too, and explains what makes them pleasing to the human eye (brain). The students gather inspiration from natural shapes and artworks to create their own designs incorporating the principles of harmony. For this purpose, they can create vision boards (on paper or digital creative boards, for example in Canva.com) to help them develop better their artistic ideas and get inspired. Step 4. Getting creative. The students plan and develop their art projects connected to the golden, silver or other metallic ratios guided by the art teacher

and the professional graphic designer. The designer introduces the students to the steps involving the realization of such a design project for a simple logo or digital artwork design – from sketching of the main idea to drawing it with a computer software.

Step 5. Exhibition of students' artworks and evaluation. The students present their artworks on an exhibition and share with their teachers and peers how they incorporated the principles of harmony and what inspired them to create their works. Each teacher follows the assessment level methodology, i.e. assesses students' knowledge, presentation and communication skills. The students can vote on the best design.

^{*} under development the final elements of the framework

3. Objectives and Methodologies

Learning Goals and Objectives	 After completing the training, the students should know: How harmony is defined in Mathematics. What the golden ratio is, basic information about the Fibonacci numbers and the golden spiral and how they are present in various natural shapes. How the golden ratio can be generalized to obtain other ratios, the so called metallic ratios. Which geometrical figures contain these ratios – golden triangles, regular polygons and others. Geometrical methods for division of a linear segment in golden ratio. How mathematical principles govern what looks pleasing to the human eye (brain) – golden ratio in the works of art: paintings, sculptures and buildings, from ancient times to present day digital art. How harmony is defined in music theory. What are the basic rules of Western musical harmony and how it influences our emotions. How mathematical principles help us understand what sounds pleasing to the human ear (brain). How graphic design software work and how it can be used for creating simple logo and digital artwork design.
Learning Outcomes and expected Results	 Students understand the basic mathematical principles of harmony and where they are found in the living nature and famous artworks. Acquisition of basic skills for computer graphic design software. After completing the lessons, the students will be able to: To understand and explain the basic mathematical principles of harmony. To explain how mathematical theory of harmony works in nature, visual arts and music. To analyze works of art for the presence of the golden ratio or silver ratio. To create and use vision boards to help them develop an artistic idea. To draw inspiration from nature and works of art. To create a simple logo or other digital artwork with a graphic design software.
Prior Knowledge and Prerequisites	 The students should be able to: To solve quadratic equations and be acquainted with irrational numbers and continued fractions. To possess knowledge of basic geometrical figures – triangles and regular polygons. To be creative and generate new ideas.

- To present to an audience.

Expected results:

- Better and deeper understanding of basic mathematical ideas of harmony and their connection to biology and the arts.
- Digital artworks dedicated to the golden ratio or silver ratio.
- Developing and enhancing of mathematical knowledge and aesthetic perceptions.

Motivation, Methodology, Strategies, Scaffolds One major task of this plan is to develop deeper understanding of the important mathematical concept of harmony, mainly through learning about the golden ratio, and how it can be found in various forms of the living nature, including the human body, as well in famous artworks – paintings, sculptures, architecture and etc. These lessons aim to enhance students' mathematical knowledge and to underline the pivotal role which mathematics plays in exploring, understanding and shaping the world around us. The other major goal of these lessons is to enhance students' creative thinking, aesthetics and artistic abilities, and to inspire them to search for hidden connections between mathematics and other subjects. Mathematics as a source of artistic inspiration. The methods used include implementing of intra-subject connections between algebra and geometry, as well as interdisciplinary connections between mathematics, natural sciences (biology) and the arts. The students will create their own works of art applying the mathematical principles of harmony by help of computer software (IT). The main motivation of this plan is to use Mathematics to inspire students to be more creative and to show them how beautiful Math is.

4. Preparation and Means

Preparation, Space Setting, <i>Troubleshooting</i> <i>Tips</i>	At different stages of the work, teachers change their leading role. In the initial period, the lead teacher is the math teacher who presents the theoretical knowledge about harmony in geometry and algebra. She/he gives various math problems to the students and facilitates them in finding the right solutions. At the next stage, the biology and the art teacher (and also the music teacher, optionally) play the key role by explaining to the students how mathematical concepts of harmony are applied in their subjects. All teachers (each according to their competencies) collaborate with the students in solving their problems, thus demonstrating the interdisciplinary nature of project-based learning. Instructional sources, digital and paper material with the related references needed for the implementation of the learning plan are used.
Resources, Tools, Material, Attachments, Equipment	 Students work in the classroom and in a computer lab while acquiring new knowledge and skills. They discuss together as a team in a STEAM center or other secure environment with their teachers. Teachers should have appropriate learning resources such as presentations, videos, practical examples, etc. Some materials and videos which can be used for initial motivation of students on the subject are the following: Golden ratio in art and nature https://www.thecollector.com/what-is-the-golden-ratio-and-how-does-it-apply-to-art/ Golden ratio in art https://blog.artsper.com/en/a-closer-look/golden-ratio-in-art/ PBS. The Golden Ratio: Myth or Math? https://www.youtube.com/watch?v=1Jj-sJ7806M The Magic of the Fibonacci numbers – Why we learn Mathematics? https://www.youtube.com/watch?v=SjSHVDfXHQ4

	 Mario Livio, The golden ratio and aesthetics, 		
	https://plus.maths.org/content/golden-ratio-and-aesthetics		
	 Metallic numbers: Bevond the aolden ratio: 		
	Part 1: https://plus.maths.ora/content/silver-ratio		
	Part 2: https://plus.maths.org/content/part-ii		
	 Dann Dassaia, Variations on a Thoma of the Silver Patio 		
	Dunn Passoja, vanations on a Thenle of the Silver Ratio		
	nttps://www.researcngate.net/publication/288496866_Variations_on_a		
	<u>_Ineme_of_the_Silver_Ratio</u>		
	• Music from the Fibonacci numbers		
	https://www.voutube.com/watch?v=IGJeGOw8TzO		
	Harmony in music – https://www.youtube.com/watch?y=KwBHu8T1ICs		
	The teachers also use the references on the first name of this night as well as:		
	Communication and collaboration platforms. Coople Most Coople		
	Communication and conduction platforms - Google Meet, Google		
	Classroom, Zoom, Skype, etc.		
	• E-learning platform - Google classroom, Moodle, etc.		
Health and Safety	Students and teachers work in a nealthy and safe environment.		
5. Implementation			
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Instructional Activities,	This plan is developed with an emphasis on classes in Mathematics, IT, Biology,		
Procedures, Reflections	Arts or in a STEAME interest club. Covers the subjects of study:		
	- Mathematics		
	- Biology		
	- Art, Digital Art, Graphic Design		
	- Presentation and communication skills		
	Teachers plan their activities in the Google environment using Google Calendar and Google Classroom as part of the curriculum. Students are actively engaged		
	through hands-on experience and research conducted as independent work that		
	can be discussed in class.		
	There are 18 study hours based on a 40-minute lesson. All classes are held once		
	or twice a week with a curriculum for 9-18 consecutive weeks.		
	- 3 hours for mathematical theory of harmony		
	- 2 hours for harmony in the arts – the golden and silver ratio in famous works of		
	art		
	- 1 hour for golden ratio in shapes of living organisms (biology)		
	- 2 hours for musical harmony		
	- 1 hour for visiting an art gallery or a museum		
	- 3 nours for learning the basics of a graphic design software		
	- 2 nours for meeting with a professional graphic designer, visiting a graphic		
	uesign studio		
	- 3 nours for creating a simple logo or algital artwork inspired by the golden		
	- 1 hour for exhibition of the students' artworks		
Assessment - Evaluation	The presentation of the final results takes place in front of the teachers, the		
	professional araphic desianer and class mates on a school exhibition where each		
	student presents the digital artwork created by them and explains how it reflects		
	the principles of harmony learned from math, biology and art classes and where		
	they got their inspiration from. Each presentation is evaluated by the teachers		
	and the graphic designer. Key factors are: student's theoretical knowledge. depth		
	of knowledge, application of theoretical concepts, communication and		
	presentation skills and aesthetics of the design.		

Presentation - Reporting - Sharing	The students' completed projects are printed on paper and presented by them to the teachers and their peers on a school exhibition. Students' artworks in digital format are published on the school website and social media accounts. The designs can be used be teachers as examples for other art projects.
Extensions - Other Information	

In the case of learning through project-based activity

STEAME ACADEMY Prototype/Guide for Learning & Creativity Approach

Action Plan Formulation

Major steps in the STEAME learning approach:

STAGE I: Preparation by one or more teachers

1. Formulating initial thoughts on the thematic sectors/areas to be covered:

Two major real life problems have driven the development of Mathematics throughout the history of human civilization – counting which helped the development of number systems, arithmetic and algebra, and measurement of quantities (perimeters, areas, volumes and etc.) which is responsible for the development of geometry. But there is a third important problem in Mathematics which also reflects it significance in natural sciences (biology) and the arts – the mathematical theory of harmony. In Math, harmony is understood as equality or proportionality of the parts between themselves and the parts with the whole. Or more general, the harmony of an object is a proportionality of the parts and the whole, a merge of the various components of the object to create a uniform organic whole. The aim of these lessons is to introduce students to the concept of harmony in Mathematics, its historical development, and to explain how it finds applications in biology, music and the visual arts. The mathematical theory focuses mainly on the golden ratio and its generalizations, the so called metallic means, defined by the Argentinian mathematician Vera W. de Spinadel in 1998, of which the silver ratio is the most popular and found in artworks and architecture. In geometry the connections between the golden and the silver ratio and certain triangles and regular polygons is explored, and in algebra the connections between these two ratios and the Fibonacci and Pell numbers, resp.

2. Engaging the world of the wider environment / work / business / parents / society / environment/ ethics:

A professional graphic designer is planned to be involved as a mentor/teacher. Visits to art galleries, museums and a graphic design studio are planned as outside the classroom activities.

- **3.** Target Age Group of Students Associating with the Official Curriculum Setting Goals and Objectives The theme is intended for students in grades 8-9 of secondary school. In Math classes in the 8th grade (in Bulgarian secondary schools), the students first learn about irrational numbers (square roots) and then are introduced to the quadratic equation and the Brahmagupta formula for finding its solutions. As the golden ratio and the other metallic ratios are positive roots of square equations, this material is suitable for 8th graders. Very basic knowledge of computer graphics and skills for working with image files are taught in the 6th grade (Bulgarian school) but this knowledge needs to be expanded for the purpose of these lessons.
- 4. Organization of the tasks of the parties involved Designation of Coordinator Workplaces etc. The teachers organize the training and support the work of the students; they motivate the students and set a real task to fulfill; the school management supports the organization of meetings with graphic designers, the extracurricular organization of the work, as well as the presentation of the results to an appropriate audience. The Math teacher can play the role of the coordinator. Workplaces to be used are a classroom and a computer lab.

STAGE II: Action Plan Formulation (Steps 1-18)

Preparation (by teachers)

1. Relation to the Real World – Reflection

Various shapes of living organisms contain the golden ratio, Fibonacci numbers and the golden spiral. The so called "divine proportion" by Luca Pacioli can be found in the human body, too. The golden and silver ratios are incorporated in famous works of art – paintings, sculptures, buildings and etc.

2. Incentive – Motivation

The math, biology, art and music teachers introduce the students to the concept of harmony in their subjects which are connected to the mathematical theory of harmony postulated by Pythagoras and Euclid. A professional graphic designer guides the students in the process of creating their own art project based on the golden, silver or other metallic ratio and the principles of harmony in art and mathematics. The students are motivated by the search of harmony in the world around us created my man or by nature. In their creative endeavor they are inspired my mathematical knowledge.

3. Formulation of a problem (possibly in stages or phases) resulting from the above

The students are first introduced to the concept of harmony by the math teacher who can show them images of flowers and animals, works of art and buildings and ask them whether they find the depicted shapes pleasing to the eye and if so, then why. The teacher can remind the students about proportions. Thus, introducing them to the golden ratio and then to its geometric and algebraic definition. Another question which the teacher can ask before beginning with the math lesson is whether the students think that mathematics has something to do with aesthetics.

Development (by students) – Guidance & Evaluation (in 9-11, by teachers)

4. Background Creation - Search / Gather Information:

New knowledge is applied when solving problems. The students are encouraged to search by themselves for information and sources of inspiration for the final creative project. The students can create real or digital vision (creative) boards with the images and information they found on the Internet.

- **5. Simplify the issue Configure the problem with a limited number of requirements** *The task is clearly stated with the necessary information.*
- 6. Case Making Designing identifying materials for building / development / creation The task that the students receive is clearly defined.
- **7. Construction Workflow Implementation of projects** Introductory training with relevant examples - Posing a real problem - Additional training -Finding a solution to the problem - Presenting the results
- 8. Observation-Experimentation Initial Conclusions The students solve math problems guided by the math teacher and experiment with creative ideas guided by the art teacher and a professional graphic designer.
- 9. Documentation Searching Thematic Areas (AI fields) related to the subject under study Explanation based on Existing Theories and / or Empirical Results Students have the necessary theoretical information and examples.
- **10. Gathering of results / information based on points 7, 8, 9** At each step, the teachers report the progress of the students.
- 11. First group presentation by students

Students present the results of their work on a school exhibition and explain the principles of harmony which they applied to create it.

Configuration & Results (by students) – Guidance & Evaluation (by teachers)

- 12. Configure STEAME models to describe / represent / illustrate the results
- 13. Studying the results in 9 and drawing conclusions, using 12
- 14. Applications in Everyday Life Suggestions for Developing 9 (Entrepreneurship SIL Days)

Review (by teachers)

15. Review the problem and review it under more demanding conditions

Project Completion (by students) – Guidance & Evaluation (by teachers)

16. Repeat steps 5 through 11 with additional or new requirements as formulated in 15

17. Investigation - Case Studies - Expansion - New Theories - Testing New Conclusions

18. Presentation of Conclusions - Communication Tactics.

STAGE III: STEAME ACADEMY Actions and Cooperation in Creative Projects for school students

Title of Project: Harmony – the Beauty of Mathematics

Brief Description/Outline of Organizational Arrangements / Responsibilities for Action

STAGE	Activities/Steps	Activities /Steps	Activities /Steps
	Teacher 1(T1)	By Students	Teacher 2 (T2)
	Cooperation with T2, T3	Age Group: 14-15	Cooperation with T1, T3 and
	and student guidance		student guidance
А	Preparation of steps 1,2,3		Cooperation in step 1,2,3
В	Guidance in step 9	4,5,6,7,8,9,10	Support guidance in step 9
С	Creative Evaluation	11	Creative Evaluation
D	Guidance	12	Guidance
E	Guidance	13 (9+12)	Guidance
F	Organization (SIL)	14	Organization (SIL)
	STEAME in Life	Meeting with Business representatives	STEAME in Life
G	Preparation of step 15		Cooperation in step 15
Н	Guidance	16 (repetition 5-11)	Support Guidance
I	Guidance	17	Support Guidance
К	Creative Evaluation	18	Creative Evaluation