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**STEAME ACADEMY**  
**TEACHING FACILITATION LEARNING & CREATIVITY PLAN (L&C PLAN) –**  
**LEVEL 2 SERVICE TEACHERS: Using Simple Linear programming in the process of**  
**looking for optimum solutions in entrepreneurial activities**

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**1. Overview**

Title	Using Simple Linear programming in the process of looking for optimum solutions in entrepreneurial activities.	
Driving Question or Topic	<p>The following guiding/ driving questions are setting the framework that will form the basic ideas of the plan</p> <ul style="list-style-type: none"> <li>• What is the problem or need that an entrepreneur is trying to solve or address in a context providing optimum solutions?</li> <li>• Who are your target customers or beneficiaries, and what are their main points or goals?</li> <li>• What are the assumptions or hypotheses that you have about your problem, solution, and customers or beneficiaries?</li> <li>• What are the main concepts and terms of linear programming, such as objective function, constraints, feasible region, and optimal solution?</li> <li>• How can linear programming be used to model real-world situations that involve maximizing or minimizing a quantity, such as profit, cost, or production?</li> <li>• How can linear programming problems be solved graphically by finding the vertices of the feasible region and evaluating the objective function at each vertex?</li> <li>• How can linear programming problems be solved algebraically by using the simplex method or other algorithms?</li> <li>• What are some examples of entrepreneurial activities that can benefit from using linear programming, such as product mix, transportation, or inventory management?</li> <li>• Can you work on a project where you apply linear programming to optimize a real or simulated entrepreneurial activity?</li> <li>• How does this project help in understand the practical application of linear programming in entrepreneurship?</li> </ul>	
Ages, Grades, ...	14-17 years old	8-11

Duration, Timeline, Activities	10 hours	3-6 activities
Curriculum Alignment	<p>The above questions imply that the whole approach concerns mainly Linear Programming and how it can be exploited in handling problems looking for the identification of optimum solutions; This quest obviously is immediately related to a broad range of mathematical concepts and processes that are the object of every day mathematical curriculum (such as functions, algebraic equations etc). Moreover, it is an essential tool in entrepreneurial activities having to do with the real world. These activities could cover content and processes relating not only to Economics but also Science, Technology and Engineering.</p>	
Contributors, Partners	<p>In the context of the consideration of this topic and taking in mind the driving questions, it is going to be useful to include the cooperation of a number of experts/ teachers covering a broad spectrum of the realms of meaning. Thus, it is suggested to involve a mathematics teacher (T1), a Science teacher (T2) and an Economics teacher (T3). Furthermore, it is going to be useful to come in contact with entrepreneurs in the real world aiming to identify issues of interest to them that are reflecting the idea of looking for optimum solutions to activities that can be represented in a context that can be modelled through linear programming</p>	
Abstract - Synopsis	<p><b><i>Using Simple Linear programming in the process of looking for optimum solutions in entrepreneurial activities</i></b></p> <p><i>This topic and the respective L&amp;C Plan are aiming at the involvement of students in the linear programming approach in the context of entrepreneurial activities. Thus students will be expected to apply mathematical reasoning and problem-solving skills to real-world situations involving optimization. Furthermore they should be able to demonstrate their understanding of linear programming concepts and methods by creating and presenting their own linear programming models and solutions, particularly in a STEAME context. In the context of the consideration of this topic, it is suggested the development of an approach that motivates students to learn about simple linear programming, by helping them to realize how this technique can help them make better decisions in various entrepreneurial activities. With this in mind there will be provision of examples involving the concepts that are essential in the context of linear programming, and development of activities that help students on working on them.</i></p> <p><i>The students are expected to be involved in project activities that will provide the opportunity for interest development, provision of concrete examples, analysis of the problem's constituents, development of a plan for solution, implementation of the plan, investigation, reflection and assessment of the outcomes.</i></p>	
References, Acknowledgements	<p>There is ample literature on the topic but the students can give emphasis on: Their textbooks on Mathematics and other areas of STEAME with chapters on activities related to optimization using linear programming approaches</p> <p>WEBSITES particularly the following</p> <p>Linear Programming (Read)   Algebra   CK-12 Foundation (ck12.org): <a href="https://www.ck12.org/algebra/Linear-Programming/lesson/Linear-Programming-ALG-I/">https://www.ck12.org/algebra/Linear-Programming/lesson/Linear-Programming-ALG-I/</a></p> <p>Linear programming Facts for Kids (kiddle.co): <a href="https://kids.kiddle.co/Linear_programming">https://kids.kiddle.co/Linear_programming</a></p> <p>Entrepreneurship for Kids: From Lemonade Stand to Startup Empire   Lemonade Day: <a href="https://lemonadeday.org/blog/entrepreneurship-for-kids">https://lemonadeday.org/blog/entrepreneurship-for-kids</a></p> <p>Platforms like YouTube or educational channels such as TED-Ed or CrashCourse Kids may have relevant videos on basic programming and optimization.</p>	

Organizations like the National Council of Teachers of Mathematics (NCTM) or local educational associations offering workshops or curriculum materials that can support the teaching efforts.

## 2. STEAME ACADEMY Framework\*

Teachers' Cooperation	<p>Teachers' cooperation would cover:</p> <p>Identification of the learning objectives and outcomes for the topic. (For example, students should be able to formulate a linear programming problem, graph the feasible region, find the optimal solution, and interpret the results in a real-world context).</p> <p>Choosing a suitable pedagogical approach and instructional strategy for the topic. (For example, teachers can use a problem-based learning approach, where they present students with a realistic and engaging problem that requires linear programming to solve).</p> <p>Deciding on what aspects each of them will have the main responsibility in helping students (For example T1 (math teacher) would concentrate on the mathematical aspects, T2 (science teacher) and T3 (Economics teacher) would concentrate on the activities covering the application/ real world issue, providing the necessary guidance to the pupils on the identification of the problem and its aspects that lead to an optimization process. Furthermore, they would support students in developing entrepreneurial structures in the context of the school.</p> <p>A fourth teacher T4 (IT or technology teacher) could cooperate with the others in helping students to use visualization and presentation material and computer programs for handling the various parameters that are involved in the problem.</p> <p>Finally, all teachers would be involved in the assessment, exploitation and reflection on the outcomes of the whole approach.</p>
STEAME in Life (SiL) Organization	<p>Through the exchange of ideas with real life entrepreneurs on aspects requiring optimization and by asking them to comment on the outcomes and the presentations of the pupils, feedback can be provided to them reflecting real life situations and in various areas stemming out of STEAME.</p> <p>Furthermore, experts from the real life can comment productively on ideas/ activities of the students that lead to implementation by them of a process aiming at optimization of a process (e.g. business or experiment or construction) developed and studied by them.</p>
Action Plan Formulation	<p>The teachers should meet at the initial stages and identify the basic aspects that are needed for the study of climate change and its repercussions on real life. Furthermore, they should exchange ideas with an expert on the field and identify actions that could be taken as a result of the consideration of the data in real life situations. Based on these they proceed to the Action Plan Formulation</p> <p>Action Plan Formulation</p> <p>STAGE I: Preparation by field one or more teachers [STEPS 1-4], and</p> <p>STAGE II: Action Plan Formulation [Preparation STEPS 1-3]</p> <p>Refers to the creation of this Learning Plan, by teachers in collaboration.</p> <p>STAGE III: Action Plan Formulation [Development STEPS 4-18]</p> <p>Refers to the realization by the students of the various activities of the Learning Plan.</p> <p>The support, feedback and evaluation by the teachers are accompanied throughout the implementation of the activities.</p>

\* under development the final elements of the framework

## 3. Objectives and Methodologies

Learning Goals and Objectives	<ul style="list-style-type: none"> <li>● To understand the basic concepts and terminology of linear programming, such as objective function, constraints, feasible region, optimal solution, etc.</li> <li>● To learn how to formulate a linear programming problem from a real-life situation, such as maximizing profit, minimizing cost, or allocating resources efficiently.</li> <li>● To learn how to graph a system of linear inequalities and identify the feasible region and the optimal solution using the corner-point method or the graphical method.</li> <li>● To learn how to use software tools, such as GEOGEBRA, to solve linear programming problems and visualize the results.</li> <li>● To apply linear programming to various entrepreneurial activities, such as product mix, transportation, scheduling, inventory, etc., and analyze the optimal solutions and their sensitivity to changes in the parameters.</li> <li>● To recognize that linear programming has many practical uses in various fields such as business, economics, engineering, operational research etc and reflect on further exploitation of it in innovative areas of real world.</li> </ul>
Learning Outcomes and expected Results	<p>Students will be able to apply mathematical reasoning and problem-solving skills to real-world situations involving optimization.</p> <p>Students will be able to demonstrate their understanding of linear programming concepts and methods by creating and presenting their own linear programming models and solutions, particularly in a STEAME context</p>
Prior Knowledge and Prerequisites	<p>Basic algebra and arithmetic skills, such as solving linear equations, inequalities, and systems of equations, and performing operations with fractions, decimals, and percentages.</p> <p>Basic geometry skills, such as finding the area and perimeter of polygons, and plotting points and lines on a coordinate plane.</p> <p>Basic logic and reasoning skills, such as identifying assumptions, variables, and constraints, and making valid arguments and conclusions.</p> <p>Basic computer skills, such as using a spreadsheet, a calculator, or a programming language to perform calculations and data analysis.</p>
Motivation, Methodology, Strategies, Scaffolds	<p><b>Motivation:</b> To motivate students to learn about simple linear programming, an approach can be based on helping them to realize how this technique can help them make better decisions in various entrepreneurial activities, such as product mix, transportation, scheduling, inventory, etc. It is also possible to identify real-world examples and case studies that illustrate the benefits and challenges of using linear programming in different contexts.</p> <p><b>Methodology:</b> Provide examples involving the concepts that are essential in the context of linear programming, and develop activities that help students on working on them and come to conclusions justifying optimal outcomes. Extend this approach in a broad range of real-world cases.</p> <p><b>Strategies:</b> To help students master and apply simple linear programming, one can use various strategies, such as:</p> <p>Providing feedback and guidance on their solutions and interpretations of linear programming problems.</p> <p>Using different types and levels of exercises to assess and reinforce their understanding and skills.</p> <p>Using cooperative learning and peer review to foster collaboration and communication among students.</p> <p>Using project-based learning and problem-based learning to engage students in authentic and meaningful tasks that require linear programming.</p>

#### 4. Preparation and Means

##### Preparation, Space Setting, Troubleshooting Tips

**Preparation and Means:** It is useful to review the basics of linear inequalities, systems of linear inequalities, and graphing linear inequalities with the students. Furthermore, prepare real-life examples of entrepreneurial activities, such as selling products, planning a budget, or allocating resources, to make the topic more relevant and interesting for the students.

Tools, such as GeoGebra, are expected to help the students visualize and explore the graphs of linear programming problems.

**Space Setting:** The classroom is going to be useful to be arranged in a way that facilitates group work and discussion, as well as individual practice. The students can be divided into small groups and assign them different linear programming problems to solve. A projector or a smart board can be useful tools to display the graphs of the problems and the solutions.

##### Resources, Tools, Material, Attachments, Equipment

**Resources:** Further to the resources already suggested in Section 1, the students may be prompted to search the web and identify examples, and practice questions on linear programming. These resources can help them understand and design their work. Another example of such resource is: <https://www.nagwa.com/en/plans/376179505956/>

**Tools:** Online graphing calculators and software, such as Desmos or GeoGebra, is quite helpful to students to visualize and explore the graphs of linear programming problems

**Material:** Worksheets, blank graph sheets, and pens or pencils can become useful companions to students to practice solving linear programming problems. In this context the use of real-life scenarios, such as selling products, planning a budget, or allocating resources, can make the topic more relevant and interesting for the students.

**Attachments:** the use of a projector or a smart board to display the graphs of the problems and the solutions are extremely useful. These devices can also be used to show videos or animations that explain the concepts and applications of linear programming.

**Equipment:** The availability of computers or tablets with internet access is obviously a useful support in a contemporary class, particularly helpful for animation activities

##### Health and Safety

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#### 5. Implementation

##### Instructional Activities, Procedures, Reflections

##### Activity 1: INTEREST DEVELOPMENT

Students are always interested about excursions. Suggest to them that the school has secured an amount of money for visiting two cities A and B, that can provide many opportunities for a broad range of activities (cultural, shopping etc). Ask the students to suggest what they would like to do in case of visiting the cities and what are some parameters that they and the school have to consider in order to secure the optimum use of the available money. With this in mind they have the opportunity of thinking about what information is needed that will help for reaching a decision on how to plan their trips.

##### Activity 2: PROVIDE A CONCRETE FRAMEWORK THROUGH AN EXAMPLE

Consider the following problem

The school wants to organize two trips for its students to visit two different cities. These two cities offer very interesting events/ activities ranging from museums,

athletic events, cultural monuments etc. The school has a fixed budget of at most 1000 euro for each student and a limited number of at most 6 days for staying in the two cities. The school wants to maximize the educational and cultural benefits of the two trips (to city A and city B), while also ensuring that the students have enough time to enjoy the attractions and activities in each city. It is given that that

- (a) The cost for staying in city A is 100 euro per day and in city B is 70 euro per day.
- (b) Travelling to city A costs 200 euro and to city B costs 300 euro. Once they go to a city the students will stay there for the whole period of activities/ visits in this city and then return back to their place, so that the next day will visit the other city or go back to school.
- (c) In city A the students can be involved in at most 6 activities per day (going to cultural events, museums, athletic events, etc.) while in city B the students can be involved in at most 5 activities per day
- (d) In city A there are 30 activities (museums, etc.) worth of spending the time attending/ visiting them while in city B there are 25 such events.

Using linear programming find the optimum number of days that have to be spend in each city so that the students enjoy the maximum number of activities.

**Activity 3: Analysis of the problem's constituents – Understand the problem.** In particular it is expected to identify the various elements/quantities that are involved in the process

- i. The variables that have to be considered
- ii. The Objective function that has to be optimized (Maximized or minimized)
- iii. Other parameters/ constraints that play important role in the next steps

**Activity 4: Develop a plan for the solution**

The plan involves the identification of mathematical relations/ models that are representations of the various concepts and consideration/ decision of mathematical approaches that were used in similar cases (eg if the representations are leading to linear relations to use a graphical method or the Simplex method or other methods) depending on the background of the students, In this case it is suggested to adopt the graphical method

**Activity 5: Carry out the plan** to Implement the previous thoughts as presented in Activity 4. In this case software for graphical representation is going to be needed. Based on the manipulation of the relations the students are expected to produce a solution.

**Activity 6: Look Back, Investigate the outcomes, assess and reflect on them.** The solution found in Activity 5 is assessed/ investigated for securing logical and correct solution

Assessment - Evaluation

The students are given, from their textbooks, similar problems to be solved either in the class or as homework  
During the processes the students are driven to discussion and reflection both on the approaches as well as on the plausibility of the solution,

Presentation - Reporting  
- Sharing

The students are asked to present their work either from projects or from solutions of their homework as in the example in the APPENDIX

Extensions - Other  
Information

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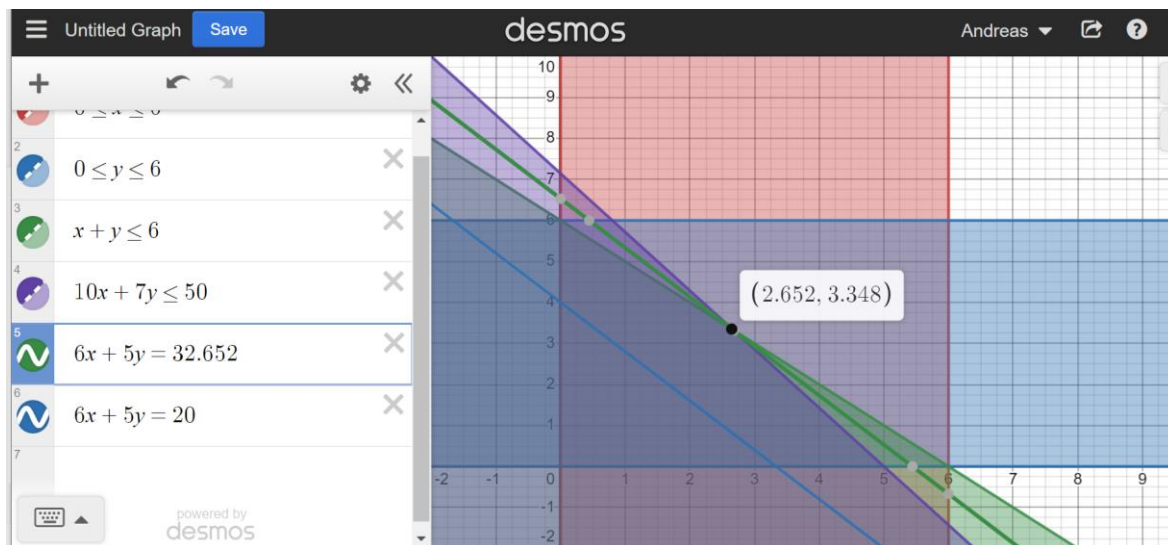


## APPENDIX the solution to the problem given in Activity 2

Let  $x$  be the number of days in city A and  $y$  be the number of days in city B

The objective function is  $z = 6x + 5y$

The constraints are  $10x + 7y \leq 50$ ,  $x + y \leq 6$ ,  $x \geq 0$ ,  $x \leq 6$ ,  $y \geq 0$ ,  $y \leq 6$



From the graph we observe that the objective function is maximum when  $x \approx 2,65$  and  $y \approx 3,34$ . But since the students should spend whole days in the cities, we conclude that  $x=2$  and  $y=3$ . Thus the maximum value for  $z=6.2+5.3=12+15=27$  and the total cost is 910 for each student.

Resources for the development of the STEAME ACADEMY Learning and Creativity Plan  
In the case of learning through project-based activity

### STEAME ACADEMY Prototype/Guide for Learning & Creativity Approach Action Plan Formulation

Major steps in the STEAME learning approach:

#### STAGE I: Preparation by one or more teachers

1. Formulating initial thoughts on the thematic sectors/areas to be covered
2. Engaging the world of the wider environment / work / business / parents / society / environment/ ethics
3. Target Age Group of Students - Associating with the Official Curriculum - Setting Goals and Objectives
4. Organization of the tasks of the parties involved - Designation of Coordinator - Workplaces etc.

#### STAGE II: Action Plan Formulation (Steps 1-18)

##### Preparation (by teachers)

1. Relation to the Real World – Reflection
2. Incentive – Motivation

3. Formulation of a problem (possibly in stages or phases) resulting from the above

Development (by students) – Guidance & Evaluation (in 9-11, by teachers)

4. Background Creation - Search / Gather Information
5. Simplify the issue - Configure the problem with a limited number of requirements
6. Case Making - Designing - identifying materials for building / development / creation
7. Construction - Workflow - Implementation of projects
8. Observation-Experimentation - Initial Conclusions
9. Documentation - Searching Thematic Areas (AI fields) related to the subject under study – Explanation based on Existing Theories and / or Empirical Results
10. Gathering of results / information based on points 7, 8, 9
11. First group presentation by students

Configuration & Results (by students) – Guidance & Evaluation (by teachers)

12. Configure STEAME models to describe / represent / illustrate the results
13. Studying the results in 9 and drawing conclusions, using 12
14. Applications in Everyday Life - Suggestions for Developing 9 (Entrepreneurship - SIL Days)

Review (by teachers)

15. Review the problem and review it under more demanding conditions

Project Completion (by students) – Guidance & Evaluation (by teachers)

16. Repeat steps 5 through 11 with additional or new requirements as formulated in 15
17. Investigation - Case Studies - Expansion - New Theories - Testing New Conclusions
18. Presentation of Conclusions - Communication Tactics.

## STAGE III: STEAME ACADEMY Actions and Cooperation in Creative Projects for school students

Title of Project: \_\_\_\_\_

Brief Description/Outline of Organizational Arrangements / Responsibilities for Action

STAGE	Activities/Steps Teacher 1(T1) Cooperation with T2 and student guidance	Activities /Steps By Students Age Group: ____	Activities /Steps Teacher 2 (T2) Cooperation with T1 and student guidance
A	Preparation of steps 1,2,3		Cooperation in step 3
B	Guidance in step 9	4,5,6,7,8,9,10	Support guidance in step 9
C	Creative Evaluation	11	Creative Evaluation
D	Guidance	12	Guidance
E	Guidance	13 (9+12)	Guidance
F	Organization (SIL) STEAME in Life	14 Meeting with Business representatives	Organization (SIL) STEAME in Life
G	Preparation of step 15		Cooperation in step 15
H	Guidance	16 (repetition 5-11)	Support Guidance
I	Guidance	17	Support Guidance
K	Creative Evaluation	18	Creative Evaluation



