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STEAME ACADEMY
TEACHING FACILITATION LEARNING & CREATIVITY PLAN (L&C PLAN) - LEVEL 1
STUDENT TEACHERS: Solving Problems with Diophantus

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1. Overview

Title	**Solving Problems with Diophantus – Modeling with linear Diophantine equations**		
Driving Question or Topic	*Who is Diophantus of Alexandria and why is he called "the father of algebra"? What is a Diophantine equation? How are linear Diophantine equation solved? How can linear Diophantine equations of two or more variables be applied to modeling and solving problems in Chemistry, Physics or Engineering (network flows), Entrepreneurship (business) and everyday life? How can IT be applied to solving linear Diophantine equations?*		
Ages, Grades, …	*12-13 years old*	*6-7 grades*	
Duration, Timeline, Activities	10 lessons	10 lessons each with a duration of 40 min.	1 lesson per week within 10 consecutive weeks
Curriculum Alignment	Mathematics, Chemistry, Physics or Engineering, Cryptography, Entrepreneurship, IT (Excel or other spreadsheet software) An important problem in number theory are linear Diophantine equations. In these lessons, the students will learn about Diophantus of Alexandria, known as "the father of algebra", and linear Diophantine equations. They will learn how linear Diophantine equations (homogenous and non-homogenous) are solved using different methods – by help of divisibility rules, the method of finding a particular solution with the extended Euclidean algorithm for the greatest common divisor (gcd) and then writing the formulae for the general solution, and using Euler's substitution method (separating integer and fractional parts in an analytical expressions). The students will discuss the number of solutions of such equations – solutions in natural numbers and in integers. They will also learn about the Frobenius coin problem and its generalizations. The students will learn how to solve linear non-homogenous Diophantine equations in Excel using the Solver function with constrains (or other spreadsheet software) or to solve such equations with a web applications. The students will aslo learn how to construct mathematical models of problems in Chemistry (balancing of chemical equations), Physics or Engineering (network flows), Entrepreneurship (business problems) and everyday life problems with linear non-homogenous Diophantine equations of two or more variables.		
Contributors, Partners	*(Optional) A professional network engineer.*		

Abstract - Synopsis	*The course begins with the mathematical theory of linear Diophantine equations. The math teacher recalls basic number theory (from 5th grade) – divisibility rules and the Euclidean algorithm for finding the greatest common divisor (gcd) of two positive integers. The math teacher can begin introducing the new material by posing to the students a simple everyday life problem which can be modeled by a linear non-homogenous Diophantine equation of two variables. The teacher guides the students in creating the model and finding the solutions to the obtained equation. The students use divisibility rules to solve the equation. Then, the math teacher introduces the students to the term "Diophantine equation", in particular to linear Diophantine equations of two variables (ax + by = c) and underlines the differences between a linear equation of one variable and an undefined multivariable equation. The math teacher introduces the students to basic methods for solving linear non-homogenous Diophantine equations: by using divisibility rules, by using the extended Euclidean algorithm for finding a particular solution and then writing the formulas for the general solution, and by the Euler's substitution method. Discusses with the students the necessary and sufficient condition for such an equation to have integer solutions, the number of integer solutions and the number of solutions in natural numbers (constrains to the variables expressed by linear inequalities). The math teacher shows also how to solve a linear Diophantine equation of more than two variables. The math teacher also introduces the students to the Frobenius coin problem and its generalizations. To the more advanced and curious students, the math teacher can explain the relation between linear Diophantine equations and congruencies, i.e. how such an equation can be expressed in the form of a linear congruence. The math teacher may also explain how linear Diophantine equations can be applied to simple problems in cryptography.* *Next, the IT teacher introduces the students to the methods which can be used for solving linear non-homogenous Diophantine equations in Excel.* *The chemistry teacher explains to the students how to balance a chemical equation which expresses a chemical reaction, and together with the math teacher explains how to apply linear non-homogenous Diophantine equations for balancing a chemical equation.* *The physics (engineering) teacher introduces the students to networks and network flows. Optionally, the school administration can invite a (network) engineer to introduce students to networks and problems involving network flow. The math teacher and the physics teacher explain to the students how linear Diophantine equations can be used for modeling a flow in a network.* *The entrepreneurship teacher poses to the students business problems which can be modeled with linear non-homogenous Diophantine equations. The students can solve the obtained equations either by hand using the methods from math classes, or in a spreadsheet software.* *The work on the subject lasts 10 hours.*
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2. STEAME ACADEMY Framework*

Teachers' Cooperation	***Teacher 1:*** *Mathematics Teacher – explains the mathematical theory of linear non-homogenous Diophantine equations of two and more variables, methods for finding their solutions and how they are applied for modeling and solving everyday problems. Introduces the students to the Frobenius coin problem.* ***Teacher 2:*** *IT Teacher – explains to the students how linear non-homogenous Diophantine equations can be solved in MS Excel spreadsheets or other similar software. Introduces the students to web applications for solving such equations.* ***Teacher 3:*** *Chemistry Teacher – Explains to the students the theory of chemical equations and how to balance a chemical equation.* ***Teacher 4:*** *Physics (Engineering) Teacher – teaches students about networks and network flows.* ***Teacher 5:*** *Entrepreneurship Teacher – introduces students to business problems which can be modeled and solved by help of linear Diophantine equations.*

STEAME in Life (SiL) Organization	*(Optional) The school administration can organize a meeting with a network engineer who can be invited to explain to the students how problems of network flow are modeled and solved.*
Action Plan Formulation	***Step 1**. **Acquisition of theoretical mathematical knowledge.** The students learn about the mathematical theory of linear non-homogenous Diophantine equations and methods for finding their solutions.* ***Step 2. Acquisition of computer skills.** The students learn how to use Excel or other spreadsheet software to solve linear Diophantine equations.* ***Step 3. Modeling and applications.** The students learn how problems in Chemistry (balancing of chemical equations), Physics or Engineering (flows in networks) and Entrepreneurship (business) can be modeled and solved by help of linear Diophantine equations.* ***Step 4. Presentation of the final project and evaluation.** Each student chooses one or more of the considered areas of applications of linear Diophantine equations – Chemistry, Engineering, business or everyday life and solves a problem in this area (or areas) by modeling it with a linear Diophantine equation. Each student solves the obtained equation by hand using one of the methods learned in math classes and by use of a spreadsheet software (or web application). The students prepare a presentation with the problem they worked on and its solution and present their projects in front of the teachers and their classmates. Each teacher follows the assessment level methodology, i.e. assesses students' knowledge, analytical skills and presentation and communication skills.*

[*] *under development the final elements of the framework*

3. Objectives and Methodologies

Learning Goals and Objectives	***After completing the training, the students should know:*** - *What a Diophantine equation is.* - *What a linear Diophantine equation is. The two basic types of linear Diophantine equations – homogenous and non-homogenous and when a non-homogenous linear Diophantine equation is solvable in integer numbers.* - *What methods are used for solving linear non-homogenous Diophantine equations.* - *How linear non-homogenous Diophantine equations are solved in positive integers in Excel or other spreadsheet program.* - *How linear non-homogenous Diophantine equations can be used for modeling problems in natural sciences, engineering, business and everyday life.*
Learning Outcomes and expected Results	*Students will possess knowledge of the methods used for solving linear Diophantine equations and will be able to solve such equations by hand and by help of MS Excel. Students will be able to model simple problems in chemistry, engineering, business and everyday by linear non-homogenous Diophantine equations of two or more variables.* ***After completing the lessons, the students will be able to:*** - *To understand and explain the basic methods for solving linear Diophantine equations.* - *To solve linear Diophantine equations in a spreadsheet software like MS Excel.* - *To model problems in chemistry (balancing chemical equations), engineering of networks (flows in networks), business and everyday life*

	by help of linear non-homogenous Diophantine equation of two or more variable.
Prior Knowledge and Prerequisites	***The students should be able to:*** - *To solve linear equations and linear inequalities of one variable.* - *To possess knowledge of divisibility rules and know how to find the greatest common divisor gcd of two positive integers by the Euclidean algorithm.* - *To have basic knowledge how to use a spreadsheet software like MS Excel (data format, data input and etc.).* - *To prepare a presentation (.ppt or other format) and present to an audience.* ***Expected results:*** - *Better and deeper understanding of principles of mathematics and mathematical knowledge.* - *Developing of analytical skills by modeling with multivariable equations and applying different methods for obtaining their solutions.* - *Developing of computer skills by using a spreadsheet software for solving equations.* - *Better understanding of how mathematics is applied to other sciences, engineering, business and that mathematical knowledge is necessary for solving everyday life problems.*
Motivation, Methodology, Strategies, Scaffolds	*One major task of this learning course is to develop the analytical skills of the students by teaching them how to model and solve problems with multivariable linear equations. These lessons aim to enhance students' mathematical knowledge and critical thinking and to underline the important role which mathematics plays in other sciences, engineering, business and everyday life. The other major goal of these lessons is to develop further the computer skills of the students by teaching them how to solve linear Diophantine equations in positive integers by using a spreadsheet. The methods used include implementing of interdisciplinary connections between mathematics, natural sciences (chemistry), engineering and entrepreneurship (business).*

4. Preparation and Means

Preparation, Space Setting, *Troubleshooting Tips*	*In the initial period, the leading teacher is the math teacher who presents the theoretical knowledge about linear Diophantine equations, basic methods for finding their solutions and applications in everyday problems. She/he gives various problems to the students and facilitates them in finding the right solutions. At the next stage, the IT teacher explains to the students how to solve linear Diophantine equations in spreadsheet software, like MS Excel. Then, the chemistry teacher, the physics or engineering teacher and the entrepreneurship teacher pose to the students problems from their subject which can be modeled by linear Diophantine equations. The students guided by the math and IT teacher model the problem with the correct equation and solve it. All teachers (each according to their competencies) collaborate with the students in solving their problems, thus demonstrating the interdisciplinary nature of project-based learning.* *Instructional sources, digital and paper material with the related references needed for the implementation of the learning plan are used.* *Students work in the classroom and in a computer lab while acquiring new knowledge and skills. They discuss together as a team in a STEAM center or other*

Resources, Tools, Material, Attachments, Equipment	*secure environment with their teachers. Teachers should have appropriate learning resources such as presentations, videos, practical examples, etc. Some materials and videos which can be used for initial motivation of students on the subject are the following:* ● *Linear Diophantine Equations, a chapter in: The Heritage of Thales by W.S. Anglin, J. Lambek, Springer, 1995.* ● *https://math.libretexts.org/Courses/Mount_Royal_University/MATH_2150%3A_Higher_Arithmetic/5%3A_Diophantine_Equations/5.1%3A_Linear_Diophantine_Equations* ● *https://www.math.uwaterloo.ca/~zcramer/MathCircles/LDE1Problems.pdf* ● *https://www.math.uwaterloo.ca/~zcramer/MathCircles/LDE2Problems.pdf* *The teachers also use the references on the first page of this plan as well as:* ● *Communication and collaboration platforms - Google Meet, Google Classroom, Zoom, Skype, etc.* ● *E-learning platform - Google classroom, Moodle, etc.*
Health and Safety	*Students and teachers work in a healthy and safe environment.*

5. Implementation

Instructional Activities, Procedures, Reflections	*This plan is developed with an emphasis on classes in Mathematics, IT, Chemistry, Physics or Engineering, Entrepreneurship or in a STEAME interest club. Covers the subjects of study:* *- Mathematics* *- Chemistry* *- Physics or Engineering* *- IT* *- Entrepreneurship* *- Presentation and communication skills* *Teachers plan their activities in the Google environment using Google Calendar and Google Classroom as part of the curriculum. Students are actively engaged through hands-on experience and research conducted as independent work that can be discussed in class.* *There are 10 study hours based on a 40-minute lesson. All classes are held once in a week with a curriculum for 10 consecutive weeks.* *- 4 hours for mathematical theory of linear non-homogenous Diophantine equations, solving everyday problems* *- 2 hours for using IT for solving linear non-homogenous Diophantine equations in Excel or other spreadsheet software* *- 1 hour for balancing chemical equations with linear non-homogenous Diophantine equations* *- 1 hour for problems involving network flows (optionally, including a meeting with a network engineer who is invited to participate in the lesson)* *- 1 hours for modeling business problems* *- 1 hour for presentation of students' projects*
Assessment - Evaluation	*The presentation of the final results takes place in front of the teachers and class mates where each student presents the problem they solved and the methods they used. Each presentation is evaluated by the teachers. Key factors are: student's theoretical knowledge, depth of knowledge, analytical thinking skills, application of theoretical concepts, communication and presentation skills.*

Presentation - Reporting - Sharing	*All presentations with the results of the work are uploaded to the school website and information is published on social media. The projects can be further developed into case studies and students and teachers can use them in their classes as teaching materials and/or be further developed as individual projects.*
Extensions - Other Information	--

STEAME ACADEMY Prototype/Guide for Learning & Creativity Approach
Action Plan Formulation

Major steps in the STEAME learning approach:

STAGE I: Preparation by one or more teachers

1. **Formulating initial thoughts on the thematic sectors/areas to be covered:**
   Linear Diophantine equations are an important and well-studied problem in number theory (called the queen of mathematics). The roots of number theory go back to the first scholars when mathematicians began to explore the fundamental properties of numbers in ancient Egypt, Babylon, Greece and etc. The first mathematicians to study number theory in ancient Greece were the members of the school of Pythagoras. Their legacy was developed further by scholars like Euclid and Diophantus of Alexandria who is known as "the father of algebra". Students familiarize themselves with basic elements of number theory, like divisibility rules and the Euclidean algorithm for finding the gcd, in the early stages of their school education. On the other hand, a variety of everyday life and business problems, and also problems in natural sciences and engineering are modeled with systems of linear equations which can be reduced to a linear Diophantine equation for two or more variables. Simplified versions of such problems can be used to demonstrate to 6th and 7th graders a variety of applications of linear non-homogenous Diophantine equations. The purpose is to create interdisciplinary connections between mathematics, on one side, and chemistry, physics or engineering and entrepreneurship, resp., on the other.
2. **Engaging the world of the wider environment / work / business / parents / society / environment/ ethics:**
   A professional network engineer can be invited to explain to the students how problems of networks flow are modeled with linear equations.
3. **Target Age Group of Students - Associating with the Official Curriculum - Setting Goals and Objectives**
   The theme is intended for students in grades 6-7 of secondary school. In Math classes, in the 5th grade (in Bulgarian schools), the students first learn the basics of number theory – divisibility rules and the Euclidean algorithm for gcd. In the 6th grade, the students first encounter the term equation and learn how to solve linear equations of one unknown variable and how to use such equations to model problems in various areas. They learn about the integer q In the IT classes, the 6th graders learn the basics of MS Excel, in Chemistry classes – how to express a chemical reaction by a chemical equation, and in the Technology and Entrepreneurship classes – how to solve simple business and money problems. Linear inequalities involving one unknown are taught in the 7th grade in Bulgarian secondary school. The goal of these lessons is to expand the knowledge of linear equations by teaching the students in the 6th and 7th grades about linear equations involving two or more variables which are solved not in whole the field of real numbers but only in positive integers (or non-negative integers). Constructing mathematical models with multivariable equations is helpful for developing the analytical skills of the students and creating and sustaining a deeper knowledge in mathematics and its connection to other subjects. As Diophantine equations are often encountered in mathematical competitions and Olympiads, including them in the curriculum can benefit the students in their preparation for participating in such contests.
4. **Organization of the tasks of the parties involved - Designation of Coordinator - Workplaces etc**.
   The teachers organize the training and support the work of the students; they motivate the students and set a real task to fulfill; the school management supports the organization of meetings with an engineer (optional), the extracurricular organization of the work, as well as the presentation of the results to an appropriate audience. The Math teacher can play the role of the coordinator. Workplaces to be used are a classroom and a computer lab.

STAGE II: Action Plan Formulation (Steps 1-18)

Preparation (by teachers)

1. **Relation to the Real World – Reflection**
   Problems of balancing chemical reactions, designing networks (network flow), cryptography, entrepreneurship (business) and everyday life can be modeled and solved by help of linear non-homogenous Diophantine equations.
2. **Incentive – Motivation**
   The math teacher introduces the students to the theory of linear Diophantine equations and methods for finding their solutions. The math teacher uses mostly examples from everyday life. The chemistry, physics (or engineering) teacher and entrepreneurship teacher pose to the students problems from their subjects which can be modeled by linear Diophantine equations. The IT teacher explains to the students how such equations can be solved using spreadsheets. The students are motivated by real life problems which they should solve by applying mathematical knowledge.
3. **Formulation of a problem (possibly in stages or phases) resulting from the above**
   The students are first introduced to an everyday problem by the math teacher which can be modeled by a linear non-homogenous Diophantine equation of two variables. The math teacher helps the students with constructing the model and solving in positive integers the obtain equation using knowledge about divisibility rules. Then, the math teacher begins with the new knowledge about what a linear Diophantine equation is and by which methods it can be solved.

Development (by students) – Guidance & Evaluation (in 9-11, by teachers)

4. **Background Creation - Search / Gather Information:**
   New knowledge applied when solving problems. The students are encouraged to search by themselves for information on the Internet and other sources. The students use this information when preparing their final projects.
5. **Simplify the issue - Configure the problem with a limited number of requirements**
   The task is clearly stated with the necessary information.
6. **Case Making - Designing - identifying materials for building / development / creation**
   The task that the students receive is clearly defined.
7. **Construction - Workflow - Implementation of projects**
   Introductory training with relevant examples - Posing a real problem - Additional training - Finding a solution to the problem - Presenting the results
8. **Observation-Experimentation - Initial Conclusions**
   The students solve problems from chemistry, engineering, business, cryptography and everyday life guided by the math and IT teacher. They learn how to model such problems with linear Diophantine equations and how to solve them by hand with the new methods they have learned and also by help of IT, in a spreadsheet software like MS Excel.
9. **Documentation - Searching Thematic Areas (AI fields) related to the subject under study – Explanation based on Existing Theories and / or Empirical Results**
   Students have the necessary theoretical information and examples.
10. **Gathering of results / information based on points 7, 8, 9**
    At each step, the teachers report the progress of the students.
11. **First group presentation by students**
    Students present the results of their work in the form of a .ppt or other presentation.

Configuration & Results (by students) – Guidance & Evaluation (by teachers)

12. **Configure STEAME models to describe / represent / illustrate the results**
13. **Studying the results in 9 and drawing conclusions, using 12**

14. Applications in Everyday Life - Suggestions for Developing 9 (Entrepreneurship - SIL Days)

Review (by teachers)

15. Review the problem and review it under more demanding conditions
Project Completion (by students) – Guidance & Evaluation (by teachers)

16. Repeat steps 5 through 11 with additional or new requirements as formulated in 15
17. Investigation - Case Studies - Expansion - New Theories - Testing New Conclusions
18. Presentation of Conclusions - Communication Tactics.

STAGE III: STEAME ACADEMY Actions and Cooperation in Creative Projects for school students

Title of Project: Solving Problems with Diophantus – Modeling with linear Diophantine equations
Brief Description/Outline of Organizational Arrangements / Responsibilities for Action

STAGE	**Activities/Steps** Teacher 1(T1) Cooperation with T2, T3, T4, T5 and student guidance	**Activities /Steps** **By Students** Age Group: 12-13	**Activities /Steps** Teacher 2 (T2) Cooperation with T1, T3, T4, T5 and student guidance
A	Preparation of steps 1,2,3		Cooperation in step 1,2,3
B	Guidance in step 9	4,5,6,7,8,9,10	Support guidance in step 9
C	Creative Evaluation	11	Creative Evaluation
D	Guidance	12	Guidance
E	Guidance	13 (9+12)	Guidance
F	Organization (SIL) STEAME in Life	14 Meeting with Business representatives	Organization (SIL) STEAME in Life
G	Preparation of step 15		Cooperation in step 15
H	Guidance	16 (repetition 5-11)	Support Guidance
I	Guidance	17	Support Guidance
K	Creative Evaluation	18	Creative Evaluation